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# Stability of a nonlinear elastic plate under lateral compression

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*Introduction.* Loss of stability and buckling of a round plate may be observed if the plate is loaded on the lateral surface. The solution to this problem is based on a bifurcation approach. In this case, a plate is considered as a nonlinear elastic body. It is important to choose the relation between stresses and deformations in sustainability problems of nonlinear elasticity. Simple laws of state (constitutive equations) were considered in early works devoted to this problem, for example, material of the "harmonic type" (Sensenig).
*Materials and Methods*. Equations of neutral equilibrium for round plates made of Murnaghan and Blatz-Ko materials are obtained. Assuming a uniform initial deformation on the plate, the stability problem is considered. Strict three-dimensional neutral equilibrium equations provide exploring related forms of equilibrium taking into account physical and geometric nonlinearity. Derivation of these equations is based on the application of the theory of superposition of small deformation on the final one.
*Results.* Progress in solution to the corresponding secular equation (with non-linear parameter entry) for practically important laws of elasticity of Murnaghan and Blatz-Ko is possible using the numerical methods only. The developed method for calculating bifurcation values of loading parameters makes it possible to analyze the effect of nonlinearity.
*Discussion and Conclusions.* The influence of physical and geometric nonlinearity on the upper critical value of the initial deformation parameter is explored. The results obtained can be used under the assessment of reliability of elastic third-order moduli for various physical materials. Data on these moduli is still scarce. The numerical research has shown that the constants given in some sources should be treated with caution. The use of elasticity moduli in the law of state of Blatz-Ko is also discussed.

***Keywords:*** finite deformation, stresses, buckling, plate, stability.

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**Introduction.** Currently, the study of new relatively simple and adequate laws of state for various materials that require considering nonlinear deformations is an actively developing area of continuum mechanics [1–10]. In the framework of the theory of superposition of small deformation on the finite one, three-dimensional equations of neutral equilibrium are derived for the materials of Murnaghan and Blatz-Ko. Based on these equations, an example of end buckling of a uniformly compressed round plate is considered [11, 12].

**Materials and Methods**

**Equations of neutral equilibrium.** Let $r, \ \varphi, \ z$ be the cylindrical coordinates of a point in an undeformed state. We assume that the initial deformation of the body is determined by the radius vector $R$:

$$\mathrm{R} = R(r)\mathrm{e}_r + dz\mathrm{i}_3, \tag{1}$$

where $R(r) = ar$, parameters $a, \ d$ are constants, $\mathrm{e}_r, \ \mathrm{e}_\varphi, \ \mathrm{i}_3$ are basis vectors.

For the coordinates of this point in the initial deformed state, we have:

$$R = ar, \quad \Phi = \varphi, \quad Z = dz.$$

Therefore, the strain gradient, the Finger strain measure, and its principal invariants are determined through the relations:

$$\nabla\mathrm{R} = \nabla\mathrm{R}^T = a(e_r e_r + e_\varphi e_\varphi) + d\, i_3 i_3, \ \ \mathrm{F} = \nabla\mathrm{R}^2,$$

$$I_1 = 2a^2 + d^2, \quad I_2 = a^4 + 2a^2 d^2, \quad I_3 = a^4 d^2. \tag{2}$$

From now on, $\nabla, \ \tilde{\nabla}$ is the nabla operator in the metric of the undeformed and initial-deformed state: $\tilde{\nabla} = \nabla\mathrm{R}^{-1} \cdot \nabla$.

We will use the neutral equilibrium equations proposed by A. I. Lurie [13]: $\tilde{\nabla}\Theta = 0$, where the tensor Θ is a linear differential operator over the vector of additional displacements W. The expressions of the components of this tensor concretized with account of the laws of state of Murnaghan and Blatz-Ko were obtained in [14, 15]. In the representations of the tensor Θ, the components are some functions defined as a result of solving the boundary value problem of the initial deformation.

**Research Results.** To consider the bending form of the plate equilibrium bifurcation, similarly to [11, 12], we accept the following additional displacement vector

$$\mathrm{W} = u(r,z)e_r + w(r,z)i_3 . \tag{3}$$

Considering (1), (2) and (3), Lurie tensor has the form:

$$\Theta = Ae_re_r + Be_\varphi e_\varphi + Ci_3i_3 + Gi_3e_r + He_ri_3 .$$

Here,

$$A = A_1\frac{u}{r} + A_2\frac{\partial u}{\partial r} + A_3\frac{\partial w}{\partial z},\; B = B_1\frac{u}{r} + B_2\frac{\partial u}{\partial r} + B_3\frac{\partial w}{\partial z},\; C = C_1\frac{u}{r} + C_2\frac{\partial u}{\partial r} + C_3\frac{\partial w}{\partial z},$$

$$G = G_1\frac{\partial u}{\partial z} + G_2\frac{\partial w}{\partial r},\; H = H_1\frac{\partial u}{\partial z} + H_2\frac{\partial w}{\partial r}.$$

We note that the structure of these operators is typical in the stability problems of cylindrical nonlinear elastic bodies. Omitting the expressions of the remaining coefficients, we give, for example, formulas for $A_1$. In the case of Murnaghan law of the state [13, 14] $A_1$ is expressed through the relation

$$A_1 = \frac{a}{d}\left(\lambda - \frac{3v_1 + 4v_2}{2} + \frac{v_1}{2}d^2 + (v_1 + 2v_2)a^2\right).$$

Henceforward, λ and μ are Lame elasticity moduli, $v_1$, $v_2$, $v_3$ are the third-order elasticity constants. For Blatz-Ko material [15]:

$$A_1 = \frac{2\mu(1-\beta)}{a\sqrt{I_3}}\left(I_3^{\lambda/2\mu} + \frac{\beta}{1-\beta}I_3^{-\lambda/2\mu}\right),$$

where β is the refining elasticity modulus.

The equation of neutral equilibrium is equivalent to the system of differential equations with respect to the components of the vector **W** :

$$\begin{cases} A_2\left(\dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} - \dfrac{u}{r^2}\right) + \left(A_3 + \dfrac{a}{d}G_2\right)\dfrac{\partial^2 w}{\partial r\partial z} + \dfrac{a}{d}G_1\dfrac{\partial^2 u}{\partial z^2} = 0, \\ \left(A_3 + H_1\right)\dfrac{\partial}{\partial z}\left(\dfrac{\partial u}{\partial r} + \dfrac{u}{r}\right) + H_2\left(\dfrac{\partial^2 w}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial w}{\partial r}\right) + \dfrac{a}{d}C_3\dfrac{\partial^2 w}{\partial z^2} = 0. \end{cases} \tag{4}$$

Assuming that the plate is loaded with uniform pressure along the lateral surface, we supplement the differential equations (4) with the equilibrium conditions at the boundary:

$$u(r,z)\big|_{r=r_n} = 0,\quad \Theta_{rz}\big|_{r=r_n} = 0.$$

At the ends, i.e., at

$$z = \pm\frac{h}{2}\;\; \Theta_{zr} = 0,\quad \Theta_{zz} = 0 . \tag{5}$$

We apply the substitution

$$\begin{cases} u = X_n(z)J_1(k_nr), \\ w = Z_n(z)J_0(k_nr). \end{cases} \tag{6}$$

Here, $n = 1, 2, \dots$, $J_0$, $J_1$ are Bessel functions of zero order and first kind, and numbers $k_nr_n$ are null functions $J_1(x)$.

Let us assume $x = k_nr$ and use the equations

$$\frac{dJ_1(k_nr)}{dr} = k_nJ_1'(x),\quad \frac{d^2J_1(k_nr)}{dr^2} = k_n^2J_1''(x),\quad \frac{dJ_0(k_nr)}{dr} = k_nJ_0'(x) = -k_nJ_1'(x),$$

and well-known identities for Bessel functions

$$J_1'' + \frac{1}{x}J_1' - \frac{1}{x^2}J_1 = -J_1,\quad J_0'' + \frac{1}{x}J_0' = -J_0 .$$

Having completed the above steps, we obtain that the variables in the differential equations (4) and under the boundary conditions (5) are separated. We arrive at the following boundary value problem for ordinary differential equations:

$$\begin{cases} X_n'' - k_n^2 \dfrac{A_2}{H_2} X_n - k_n \dfrac{G_2 + C_2}{G_1} Z_n' = 0, \\ Z_n'' - k_n^2 \dfrac{G_1}{C_3} Z_n + k_n \dfrac{G_2 + C_2}{C_3} X_n' = 0. \end{cases} \tag{7}$$

$$\begin{gathered} k_n C_1 X_n\left(\pm h/2\right) + C_3 Z_n'\left(\pm h/2\right) = 0, \\ k_n G_2 Z_n\left(\pm h/2\right) - G_1 X_n'\left(\pm h/2\right) = 0. \end{gathered} \tag{8}$$

Two homogeneous linear ordinary differential equations (7) and four boundary conditions (8) result in the eigenvalue problem with a nonlinear occurrence of the parameter. In this problem, such a parameter is the quantity $\left(1-a\right)$. The system (7) is ended in the standard form:

$$\begin{cases} y_1' = y_{2,} \\ y_2' = k_n^2 \dfrac{A_2}{H_2} y_1 + k_n \dfrac{G_2 + C_2}{G_1} y_4, \\ y_3' = y_4, \\ y_4' = k_n^2 \dfrac{G_1}{C_3} y_3 - k_n \dfrac{G_2 + C_2}{C_3} y_2. \end{cases}$$

The following notations are accepted here: argument $t \equiv z$, $\left(y_1;\, y_2;\, y_3;\, y_4\right)^T \equiv \left(X_n;\, X_n';\, Z_n;\, Z_n'\right)^T$. Let the fundamental system of solutions be the following four vectors $\mathrm{y}_i \equiv \left(y_{1i};\, y_{2i};\, y_{3i};\, y_{4i}\right)^T$. Then, the general solution to the system is given by $\left(X_n;\, X_n';\, Z_n;\, Z_n'\right)^T_{общ.} = \sum_{i=1}^{4} \xi_i \mathrm{y}_i$, where $\xi_i$ are arbitrary constants. For example, we set the initial data for $z = -h/2$ by the columns of the fourth-order unity matrix. Then we solve numerically the Cauchy problem with these initial conditions. As a result, on the right-hand side, we obtain the values of the basis functions, i.e., vectors $\mathrm{y}_i(+h/2)$. Using the boundary conditions (8), we arrive at a homogeneous system of linear algebraic equations with respect to $\xi_1$, $\xi_2$, $\xi_3$, $\xi_4$:

$$\sum_{j=1}^{4} a_{ij} \xi_j = 0, \text{ where } i = 1, 2, 3, 4.$$

Here, the coefficients are the elements of matrix $\mathrm{A}$:

$$\begin{gathered} a_{1j} = k_n C_1 y_{1j}\left(-h/2\right) + C_3 y_{4j}\left(-h/2\right),\ a_{2j} = k_n C_1 y_{1j}\left(h/2\right) + C_3 y_{4j}\left(h/2\right), \\ a_{3j} = k_n G_2 y_{3j}\left(-h/2\right) - G_1 y_{2j}\left(-h/2\right),\ a_{4j} = k_n G_2 y_{3j}\left(h/2\right) - G_1 y_{2j}\left(h/2\right). \end{gathered}$$

The homogeneous system has a nontrivial solution if the condition is met

$$\det \mathrm{A} = 0. \tag{9}$$

The determinant expression includes the load parameters $a$, $d$, Bessel null-functions, as well as the elasticity moduli $\lambda$, $\mu$, $v_1$, $v_2$, $v_3$ (for the Murnaghan material) or $\lambda$, $\mu$, $\beta$ (for the Blatz-Ko material).

The parameters $a$, $d$, which set the initial deformation, are interconnected. The axial force acting on the cross-sectional area is determined by the relation [13]: $Q = 2\pi \int_0^{R_н} \sigma_Z R dR$, where $\sigma_Z$ is the physical component of the stress tensor. Since the initial deformation is assumed in the form (1), the Cauchy stress tensor $\mathrm{T}$ and the Finger strain measure $\mathrm{F}$ are coaxial.

Moreover, the stress tensor is constant:

$$\mathrm{T} = \frac{2}{a^2 d}\left(c^{(0)}(a^2(e_r e_r + e_\varphi e_\varphi) + d^2 i_3 i_3) - c^{(1)}(a^4(e_r e_r + e_\varphi e_\varphi) + d^4 i_3 i_3) + c^{(-1)}(e_r e_r + e_\varphi e_\varphi + i_3 i_3)\right).$$

The ends of the plate are free of load, therefore $\sigma_Z = 0$. This implies the condition relating the coefficients of the Finger law

$$c^{(0)} - c^{(1)} d^2 + \frac{c^{(-1)}}{d^2} = 0. \tag{10}$$

Considering the law of state, we can write a specific expression of the condition (10), which establishes a connection between *a* and *d*. So, for the Murnaghan material, we get the condition:

$$\left(\frac{\nu_1}{4} + \frac{3v_2}{2} + 2v_3\right) d^4 + \left(\lambda + 2\mu - \frac{3v_1}{2} - 5v_2 - 4v_3 + (v_1 + 2v_2) a^2\right) d^2 -$$

$$-3\lambda-2\mu+\frac{9v_1}{4}+\frac{9v_2}{2}+\nu_3+(2\lambda-3v_1-4v_2-\frac{3v_3}{4})a^2+(v_1+v_2-\frac{15v_3}{8})a^4=0.$$

In particular, if $v_1=v_2=v_3=0$, then we get

$$d^2=1+\frac{2\lambda}{\lambda+2\mu}\left(1-a^2\right).$$

When considering the Blatz-Ko law of state, $d$ is expressed by the formula

$$d=a^{-2/3}.$$

Thus, through setting the elasticity moduli and Bessel null-functions, we find the bifurcation values of the initial strain parameter $a_*$ from (9).

Note that the value $\Lambda\equiv 1-a_*$ is a small parameter for relatively thin disks. So, in the classical theory of plate buckling, the critical value of $a_*$ is determined by the formula

$$a_*=1-\frac{(3,8317)^2}{12(1+v)}\left(\frac{h}{r_и}\right)^2,$$

where $v$ is the Poisson's ratio, and 3.8317 is the first root of the Bessel function $J_1(x)$.

When solving the initial boundary value problem, it is assumed that the Signorini’s perturbation method can be applied. At this, we assume that the coefficients of the operator $\Theta$ depend on a small parameter $\Lambda$ in a power-law manner. This means that the boundary conditions for the incremental displacement components $\Lambda$ are specified by the matrix $\mathrm{A}=\mathrm{A}(\Lambda)$. In this problem, a partial eigenvalue problem — the determination of the lowest eigenvalues — has physical meaning. The higher degrees $\Lambda$, which contain the matrix elements, slightly affect the values of the smallest roots of the secular equation (9). If we restrict ourselves to the linear theory under solving the initial problem, then $\mathrm{A}(\Lambda)$ is a regular binomial [16].

Numerical experiments under studying the stability of nonlinear elastic bodies of not too large relative thickness confirm this conclusion. Therefore, a characteristic equation with non-linear occurrence of parameter (9) can be replaced by the characteristic equation of the linear operator. Iterative processes that converge to one eigenvalue, where a number close to the value in the theory of plates is chosen as the null approximation, can be applied.

**Discussion and Conclusions.** Using the equations obtained above, a numerical analysis of the influence of physical and geometric nonlinearity on the value of the upper critical parameter $a_*$ is performed. The calculations are implemented in the Matlab environment for various options of specifying elasticity moduli, relative plate thickness, and waveformation number $n$. It is found that in all cases of loss of disk stability, the first-mode buckling that corresponds to the minimum critical value of the parameter, which corresponds to the root of the Bessel function 3.8317 [17], answers the smallest critical parameter value $a_*$.

Table 1 shows the critical parameter values $a_*$ for plates with a relative thickness of 0.05; 0.1; 0.15; 0.2; 0.25; and 0.3.

Table 1

Critical parameter values $a_*$ for plates of various relative thicknesses

| $h/r_и$ | Relative plate thickness | | | | | |
|---|---|---|---|---|---|---|
| | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 |
| 1 | 0.9922<br>0.9972 | 0.9869<br>0.9867 | 0.9800<br>0.9735 | 0.9660<br>0.9549 | 0.9530<br>0.9300 | 0.9330<br>0.8914 |
| 2 | 0.9977<br>0.9967 | 0.9908<br>0.9869 | 0.9801<br>0.9690 | 0.9664<br>0.9410 | 0.9504<br>— | 0.9332<br>— |
| 3 | 0.9976<br>0.9965 | 0.9907<br>0.9856 | 0.9800<br>0.9665 | 0.9662<br>0.9372 | 0.9502<br>— | 0.9330<br>— |
| 4 | 0.9977<br>0.9881 | 0.9907<br>— | 0.9798<br>— | 0.9659<br>0.9965 | 0.9497<br>0.9613 | 0.9324<br>0.9861 |
| 5 | 0.9914<br>0.9929 | 0.9865<br>0.9729 | 0.9783<br>0.9341 | 0.9668<br>0.9114 | 0.9520<br>— | 0.9350<br>— |
| 6 | 0.9978<br>0.9985 | 0.9912<br>0.9941 | 0.9800<br>0.9871 | 0.9675<br>0.9775 | 0.9520<br>0.9651 | 0.9340<br>0.9539 |
| 7 | 0.9985<br>0.9985 | 0.9941<br>0.9941 | 0.9859<br>0.9863 | 0.9735<br>0.9749 | 0.9560<br>0.9594 | 0.9320<br>0.9396 |

The following numbers indicate the materials listed below.

1. Steel Rex 535 ( $\lambda = 1.09$ , $\mu = 0.818$ , $\nu_1 = -1.75$ , $\nu_2 = -2.40$ , $\nu_3 = -1.69$ ).
2. Steel 50HGSM2F ( $\lambda = 1.129$ , $\mu = 0.803$ , $\nu_1 = -2.8$ , $\nu_2 = -2.7$ , $\nu_3 = -1.87$ ).
3. Steel Hecla 37 ( $\lambda = 1.11$, $\mu = 0.821$, $\nu_1 = -3.58$, $\nu_2 = -2.82$, $\nu_3 = -1.77$ ).
4. Steel Hecla ATV ( $\lambda = 0.87$, $\mu = 0.716$, $\nu_1 = 0.34$, $\nu_2 = -5.52$, $\nu_3 = -1.0$ ).
5. Beryllium bronze ( $\lambda = 1.042$, $\mu = 0.49$, $\nu_1 = -4.0$, $\nu_2 = -1.7$, $\nu_3 = -0.6$ ).
6. Organic glass ( $\lambda = 0.39$, $\mu = 0.186$, $\nu_1 = -0.078$, $\nu_2 = -0.07$, $\nu_3 = 0.047$ ).

In the first six cases, the Murnaghan law of state is considered. The last option presents the results for the material of Blatz-Ko. The top number in the table cell refers to the case in which physical non-linearity is not taken into account, i.e., $\nu_1 = \nu_2 = \nu_3 = 0$ in the Murnaghan law; $\beta = 0$ in the Blatz-Ko law. The second (lower) number takes into account physical nonlinearity. In the Murnaghan law, data were used for the third-order elasticity moduli from [14] in units $10^{12} \dfrac{dyne}{cm^2}$.

In the last row of Table 1, there are the results for the Blatz-Ko law, which was chosen in a simplified version (the Knowles-Sternberg equation): the Poisson's ratio was taken equal to 0.25, and the refining module $\beta = 0.45$ . A dash means no critical value $a_*$ was found.

The analysis of the results provides drawing some conclusions. At small relative thicknesses of the disk, the exact theory and the linear theory of plates give the same critical parameter values $a_*$. Geometric nonlinearity has marked impact at relative thicknesses greater than 0.1. Physical nonlinearity is even more pronounced. However, care should be exercised in choosing third-order elasticity moduli. For example, in the fourth version (Hecla ATV steel) and for thin plates, no reliable critical values of the parameter $a_*$ were found; although acceptable values indicating a loss of stability are observed for the same Lame elasticity moduli. As in the stability problem for a nonlinear elastic sphere made of Blatz-Ko material [15], the constant β weakly affects the critical value $a_*$.

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*The author has read and approved the final manuscript.*